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WAVES IN MEDIA CONTAINING BUBBLES

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Media with bubbles absorb acoustical waves to a great extent. They have great non-linear parameter values and complicated sound dispersion.

Measured values of nonlinear parameter γ for polyurethane foams were about 70. For water and glycerine containing gas bubbles as well as for water-saturated soils – up to 10 000. There is a simple formula that describes dependence of nonlinear parameter value for liquids containing gas bubbles and water-saturated soils upon volume gas concentration. Great γ values made it possible to investigate strong nonlinear effects with not so big wave amplitudes. The effect of waves interaction selfamplification where gas bubbles appeared in the vicinity of pumping wave transducer because of gas cavitation and increased the interaction efficiency was an interesting phenomenon too. We have also investigated periodic – nonperiodic conversion of gas bubble pulsations in the field of a pumping wave. This conversion can be realized through the classical bifurcation – multiplication of pulsation period – as well as through an immediate conversion to chaos.

Laboratory of Nonlinear Acoustics was founded in 1957 as a part of Radiophysics Chair by Georgiy Andreevich Ostroumov. It was he who decided to use the Tait equation for porous media mechanical properties description. The original experiments of V.M. Kriachko discovered nonlinear acoustical parameter values for polyurethane foams up to 70. The following experiments made by A.S.Tokman for glycerine containing air bubbles gave values of γ up to 10 000, what in three decades exceeded the same values for condensed media without bubbles. The experimental data were in a good agreement with derived formula of γ dependence upon volumic concentration of gas bubbles. The experiments on nonlinear effects for transition and reflection of shock waves on the boundary of liquid containing gas bubbles and pure liquid or solid were made too /1/. In the experiments with periodical waves propagation two interesting effects were discovered /2/. It can be possible to modulate the amplitude of sine wave, propagated through the liquid containing gas bubbles, by the strong impulse acoustical wave.

The second effect – selfamplification of parametrical interaction in the water with solved gas – was discovered for the first time in acoustical pool with three hundred tonnes of unsettled tap water. The bubbles, which appeared in the vicinity of the transducer as a result of gaseous cavitation, were increasing the media nonlinear parameter. As a result the difference frequency wave amplitude increased in some cases up to two decades.

In the computer experiments the pulsations of gas bubbles in the field of acoustical wave were investigated. It was discovered that conversion from the periodic pulsations to chaos was possible as by multiplication of pulsations period, i.e. bifurcations or suddenly, as the devil from the snuff-box.

The region of cavitation were investigated separately /3/. The values of nonlinear parameter γ about 1000 and it dependence upon the "age" of cavitation region were measured.

The investigation of nonlinear properties of liquids containing gas bubbles led us to exploration of more complicated media – water-saturated soils. This is multicomponent medium, which consists in simplest case of solid component – sand, for example, liquid component – water and a little quantity of squeezed air bubbles. The models, suggested for equations of state for soils, were very complicated and, wherefore, not fruitful.

A large number of our experiments with liquids containing gas bubbles and water-saturated soils made it clear that spectral components of acoustic waves with frequencies equal or higher than resonance frequencies of gas bubbles were dissipated to a large extent. So, only subresonance, low frequency waves can effectively propagate through water-saturated soils.

For this reason G.M.Liakhov /4/ suggested his quasistatic model for water-saturated soil which was based on the same low-frequency point of view. We shall deal with this model because of the aim of our experiments was to explore nonlinear properties of soil and to calculate the nonlinear parameter value, leaving apart dissipation effects.

Dr. Liakhov assumed water-saturated soil behaves like a liquid, i.e. that the pressure in all three (or more) components is the same. The equation of state for gas was chosen in Poisson adiabatic form

$$(\Delta P / P^*_{01} + 1) = (\rho / \rho_{01})^{\gamma_1}$$

Here P^*_{01} – initial pressure in the undisturbed medium,

ΔP – experimentally measured excess or acoustical pressure,

ρ, ρ_{01} – density and initial density, $\gamma = C_p/C_v$ The equations of state for –condensed media were chosen of the same form – P.G.Tait equation of state

$$(\Delta P / P^*_{0i} + 1) = (\rho / \rho_{0i})^{\gamma_i} .$$

Here ρ_{0i}, γ_i and P^*_{0i} – are the parameters of liquid (i = 2) and solid (i = 3) components, but it is important to pay attention to the fact that here P^*_{0i} is not the pressure and γ_i is not the C_p/C_v but are empirical coefficients only.

So the equation of state for water-saturated soil may be written in the case of three components:

$$\frac{\rho_0}{\rho} = \sum_{i=1}^{3} \alpha_i \left[\frac{\Delta P}{P_{oi}^*} + 1 \right]^{-\frac{1}{\gamma_i}} \quad . \tag{1}$$

Here α_i is the volumic concentration of component. It is easy to find dependence of shock wave velocity D upon rate of flow V with this equation of state. On shock front

$$(\Delta\rho/\rho) = (V/D) = \frac{\Delta P}{\rho_0 D^2} \quad .$$

From this formula:

$$D^2 = \left(\frac{\Delta P}{\Delta\rho}\right)\left(\frac{\rho}{\rho_0}\right) = \frac{\Delta P}{\rho_0 (1 - \rho_0/\rho)} \quad .$$

At last if made use of (1)

$$D^2 = \frac{\Delta P}{\rho_0} \left\{ 1 - \sum_{i=1}^{3} \alpha_i [\Delta P / P_{oi} + 1]^{-\frac{1}{\gamma_i}} \right\}^{-1} \quad . \tag{2}$$

As $V = \Delta P / \rho_0 D$ then

$$V^2 = \frac{\Delta P}{\rho_0} \left\{ 1 - \sum_{i=1}^{3} \alpha_i [\Delta P / P_{oi} + 1]^{-\frac{1}{\gamma_i}} \right\} \quad . \tag{3}$$

This is of course not the simple equation of state too.

Therefore G.A.Ostroumov and A.S.Tokman proposed an ingenious idea - to substitute this equation (1) at the beginning, where $\Delta\rho/\rho_0 = V/C_0 \ll 1$ with another equation of state for this mixture – in the Tait form with some effective parameters P^*_m and γ_m.

This could be done for long-wave approximation only, in which bubbles and grains sizes and distances between them are very small compared with wave length. Effective nonlinear parameter value could be naturally obtained from equation

$$\gamma_m = 1 + (\rho_{om} / C_{om}^2) \frac{\partial^2 P}{\partial \rho^2} \, .$$

And, obviously, $\gamma_m P^*_m = \rho_{om} C_{om}^2$. After doing calculations

$$\gamma_m = \frac{\sum_{i=1}^{3} \alpha_i \dfrac{\gamma_i + 1}{\gamma_i^2 P^{*2}_{oi}}}{\left(\sum_{i=1}^{3} \dfrac{\alpha_i}{\gamma_i P^*_{oi}}\right)^2} - 1 \quad . \tag{4}$$

This formula, for the first time being worked out by A.S.Tokman /1/ for liquids containing gas bubbles, was and is very useful for a lot of purposes, e.g. – all the formulas of nonlinear acoustics of gases, comprising γ value, can be applied to a water-saturated soils or a liquid containing gas bubbles. And, moreover, one could discuss solutions of Burgers equation for mixtures containing gas bubbles, including water-saturated soil, knowing exactly the nonlinear parameter value. It is easy to calculate shock front formation distance $x = C_o/(\omega\varepsilon M)$ or even shock front velocity /5/

$$D = \frac{V}{1-(1+\frac{\gamma_m - 1}{2}M)^{2/(1-\gamma_m)}}.$$

For $\gamma_m M << 1$ $D = C_o(1 + \varepsilon M/2)$. (5)

Here C_0 is linear-acoustical sound velocity, $M = V/C_0$ – Mach number, $\varepsilon = (\gamma_m + 1)/2$. It is convenient to use formula (5) for experimental determination of nonlinear parameters of mixtures containing gas bubbles.

We have carried out a lot of experiments in glycerine and water containing air or hydrogen bubbles to find the nonlinear parameter values. All experimental data /1/ were in reasonable accordance with those derived from (4).

To check the applicability of (4) not only for liquids but for water-saturated soils as well, 26 series of experiments were carried out. The main part of experimental installation was a brass shock tube, 39 cm height, 5 cm inner diameter, 9 cm outer diameter. It was checked that the plane waves were propagated along the tube. At the bottom of the tube an impulse electrodynamical transducer has been connected. The transducer consists of a flat spiral wire coil and membrane – copper disc about 1 mm thick. The disc was pushed from the coil when there was a capacitor of 200 μF discharge through the coil. The disc generated an impulse wave of pressure in the tube with amplitudes from 1000 up to $2\cdot 10^6$ Pa. The first pressure pulse had a form of positive swing of sine with duration about 50 μs.

The wide range of possible parameters of soils were covered by the correct choice of model soils. A light-weight granulated polyethylene, middle-weight ground chalk and ordinary sand being used as a solid components. Water and glycerine as the liquids. And air as a gaseous component.

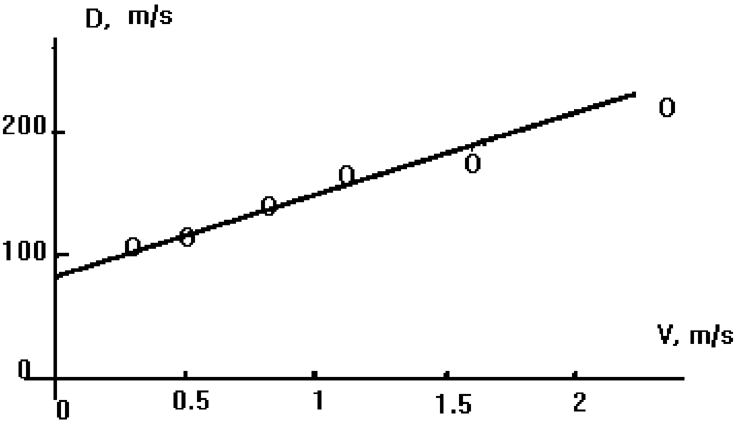
All experiments were made as follows: the shock tube was filled by a dry solid component. Then a liquid component was added until the model soil was absorbed and an excess of liquid was visible on the surface. After that the contents of the tube were mixed to get rid of the largest air bubbles.

Two self-made hydrophones, with spherical piezoelements 7 mm in diameter, were placed into the soil, on the axis of the tube, at the distances of 50 and 100 mm from the membrane. At first the capacitor was charged up to a voltage within the range of 300 – 2000 V. Then it was discharged through the transducer and the first shock wave of pressure was fixed by an oscilloscope from the first hydrophone. Then the same procedure was repeated with exactly the same voltage with the second hydrophone. Because of dissipation, the waves had different amplitudes, and an arithmetic mean was used in further calculations. The smaller the distance between hydrophones, the smaller the difference in amplitudes, but the larger error in shock wave velocity calculation.

The shock front velocity was calculated from known distance and difference of two wave arrival times. The time of arrival was calculated in such a manner: a tangent in the steepest place of the front was plotted on oscillogram and an intersection point of this one with a zero line was considered as the time to be found. This method, obvious for shock waves, was applied for calculating linear-acoustic sound velocity as well.

As a result there was shock wave velocity D and mean pressure ΔP on the shock front for two points of measuring. Keeping in mind linear dependence of D over V (5) and the possibility of obtaining γ_m value from this formula, the experimental data were represented in graphical form as D=D(V), where $V=\Delta P/\rho_o D$, ΔP – mean pressure in an experiment.

An example is shown on Fig.1.



This is dependence of shock wave velocity D on flow rate V for model soil, consisted of sand -60%, water -39%, air – 1.1%. Solid curve, here looks like straight line, was plotted by means of (2), (3) and method, described below. For this curve, linear-acoustic sound velocity is 78 m/sec, γ_m=220, $P_m=5.6\cdot10^4$ Pa and root mean square $\Delta D/D_c$ is about 2.5%.

During our early experiments a certain linear part of the D(V) dependence in small flow rates region was used to calculate γ_m value. The tangent to the curve in the place of its origin has been drawn by eye. In this case it was hard to choose the "right" set of experimental points. Then a more ingenious way was found.

With (2) and (3) one could calculate and plot a curve shown on Fig.1. Of course it is important to know the densities of solid, liquid and gaseous components to derive ρ_o and concentrations of these. For that purpose the volume of the solid component and the total volume of the fluid components have been measured. Further evaluations pointed out that there is a very week dependence of D values upon the type and concentration of liquid and solid components. It is important to draw attention to the fact that concentrations of gas were small and less than experimental errors of measured condensed media concentrations. So, this small concentration was not measured directly.

The easiest way would be to use linear-acoustical sound velocity C_0, which, of course, is a function on α_1 – concentration of gaseous component. But there are two obstacles to the successful employment of this method. The first – there is no experimental data with zero flow rate and all other D values are bigger than C_o (see(5)). The second – even if one found any values with small M enough, there is a big scattering of experimental data and α_1 value would have a big error as well.

So an original way has been used for calculating of α_I value. A computer programme has selected the α_I value, with which the result total root mean square of differences $\Delta D/D_c$ became minimal. Here $\Delta D = D_e - D_c$, D_e and D_c are the experimental and calculated with (2) – (3) shock wave velocities. For all 26 series of experiments the maximum total root mean square difference did not exceed 8%.

The main conclusion from these experiments was the following: nonlinear properties of water-saturated soil were clearly defined from a concentration of gaseous component to a considerable extent. The nonlinear parameter value depends in most cases upon this. In the case of water-saturated soils, prepared by method mentioned above, α_1 always exceeded 10^{-3} value. For these concentrations, formula (4) can be simplified up to

$$\gamma_m = \gamma_1 + 1/\alpha_1 - 1 \quad \text{or for air } \gamma_m = 2.4/\alpha_1 - 1.$$

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